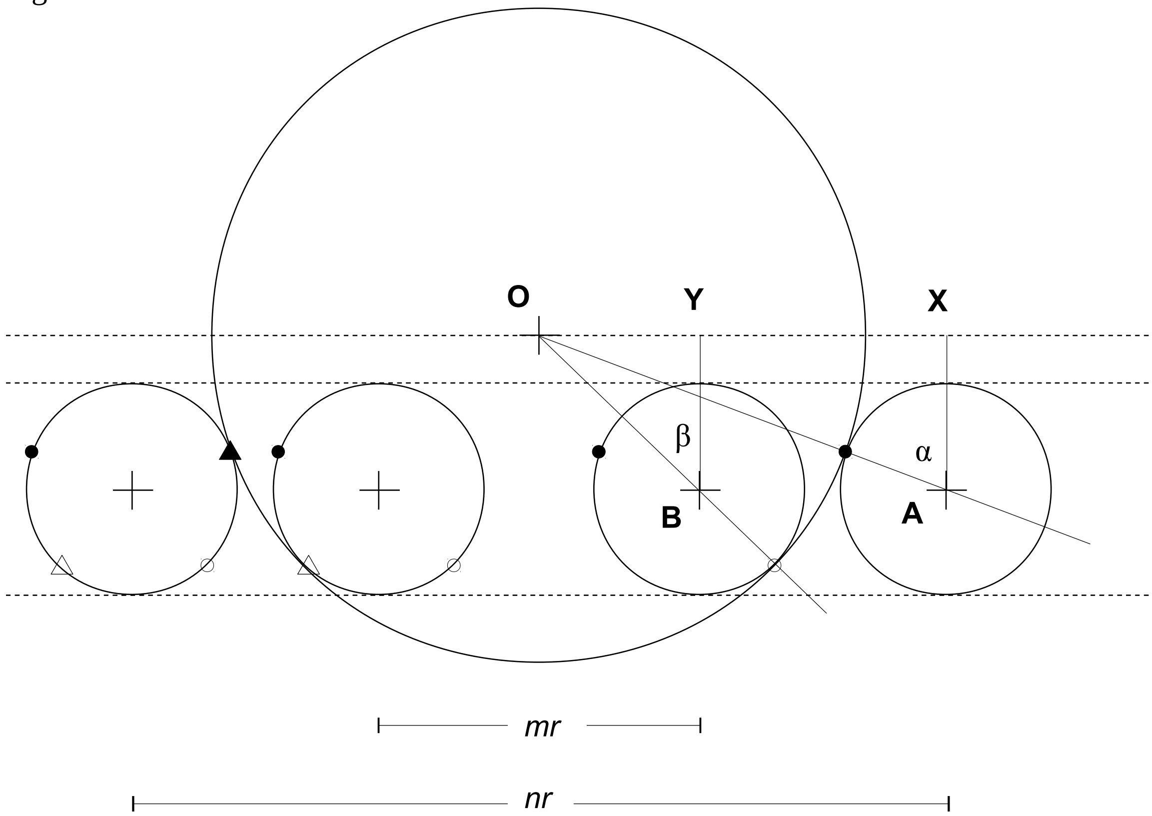
Lunar Eclipse - ANALYSIS 2

This method is suitable if you have not recorded a large number of shadow arcs, but you *have* got the timings of 1st, 2nd, 3rd and 4th contacts together with the 'clock-face' positions around the Moon's circumference where they occurred. The positions of 2nd and 3rd contact are not all that easy to determine because you can't see any of the Moon's features (particularly the **mare**, the grey areas that you would want to use to find your way around the Moon's disc). But, as we shall see, that might not matter too much.



The diagram shows the moon travelling through the shadow from right to left. The solid circle, open circle, open triangle and solid triangle show (respectively) the positions of 1st, 2nd, 3rd and 4th contacts. You will notice from the left-hand diagram of the Moon that these four contact points form a trapezium, and that the perpendicular bisector of the trapezium is the axis of the Moon. So if you hadn't previously known where the Moon's axis is, you do now!

- Draw in the Moon's axis
- Draw a diameter through the centre of the Moon and the point of 1st contact (the solid circle). The angle between these two lines is the angle α .
- Draw a diameter through the centre of the Moon and the point of 2nd contact (the open circle). The angle between these two lines is the angle β.

You can do an exactly similar procedure for the solid and open triangles, which, by symmetry, will give you second versions of α and β. (The two values of β [and of α] should, of course be equal. If they are not you can take an average.)

Now look at triangle OBY. Simple trigonometry gives

$$\cos(\beta) = \frac{adjacent}{hypotenuse} = \frac{YB}{OB}$$

We don't know YB of course, but we do know that OB is the difference between the radius *R*

Earth's shadow and the radius r of the Moon: i.e. $(R - r)$. Re-arranging the formula gives

$$YB = (R-r)\cos(\beta)$$

A similar argument applied to triangle OAX yields

$$AX = (R+r)\cos(\alpha)$$

Since AX and YB are equal, we may write

$$(R-r)\cos(\beta) = (R+r)\cos(\alpha)$$

When numerical values are plugged in for cos(α) and cos(β) we obtain a linear equation in R and r that may be solved to find the ratio R/r, which is, of course the object of the exercise.

For a variety of reasons, it is not very easy to obtain sufficiently accurate values of the two angles if one only has 1st and 2nd contacts available. However, if one has 3rd and 4th contacts available as well, then a better method is available.

Consider once again triangle OYB. We can write the formula for the sine of the angle:

$$\sin(\beta) = \frac{opposite}{hypotenuse} = \frac{OY}{OB} = \frac{\frac{1}{2}mr}{(R-r)} = \frac{mr}{2(R-r)}$$

It's the third equals sign that needs unpacking a little. We have already discussed the fact that OB = $(R - r)$, and it should be reasonably clear from the diagram that OY is half of the distance labelled as mr. Now mr is the distance travelled by the *centre* of the Moon between 2nd and 3rd contacts. Knowing how long this took, and knowing that the Moon travels its own diameter ($=2\times r$) each hour, it isn't hard to work out how many Moon radii the Moon travels between 2nd and 3rd contacts. This number of Moon radii is the m of mr. So plugging in numerical values of m and β, we obtain an equation between R and r, from which it is reasonably easy to extract a value for R/r.

The aforementioned isn't all that easy to do because of the difficulty of determining the clock-face positions of 2nd and 3rd contacts. On the other hand, the clock-face positions of 1st and 4th contacts may be determined with reasonable accuracy, so that the angle α is rather easier to find. We can then repeat the whole analysis in terms of triangle OXA, obtaining

$$\sin(\alpha) = \frac{opposite}{hypotenuse} = \frac{OX}{OA} = \frac{\frac{1}{2}nr}{(R+r)} = \frac{nr}{2(R+r)}$$

This is probably the best method of analysis. To establish that the Moon travels its own diameter each hour, you make use of the fact that the **synodic** period of the Moon is 29.5 days, so that in 29.5 days (which you can easily convert to hours) the moon travels through 360° as it goes round the Earth once. We know (I hope, by now!) that the moon subtends an angle of ½°, so it's just a question of finding how long it takes the Moon to travel that ½°. (The reason we use the synodic period rather than the **sidereal** period is to take account of the fact that the Earth's shadow is moving as well, because of the Earth moving round the Sun once a year.)